Compressive Membrane Action in Bridge Deck Slabs

Geoff Taplin *Maunsell Australia Pty Ltd*
Alan Hon *Maintenance Technology Institute, Monash University*

SYNOPSIS

Compressive membrane action is usually ignored in bridge design, and the entire load is assumed to be carried by flexure of the deck slab. Allowing for the contribution of compressive membrane action may reduce the need for strengthening or replacement of deck slabs that do not have adequate flexural strength.

This paper describes a method for assessing typical beam-and-slab bridge decks taking into account compressive membrane action. The method has been developed through the laboratory testing of concrete specimens and the use of non-linear finite element modelling. The application of the method however only requires a frame analysis program.

1 COMPRESSIVE MEMBRANE ACTION

Compressive membrane action occurs in a reinforced concrete slab subjected to vertical loads when the ends of the slab are restrained against horizontal translation. The end restraints induce in-plane compressive forces within the slab. This is referred to as compressive membrane action (or arching action) and it increases the stiffness and load-carrying capacity of the slab.

There are two requirements for compressive membrane action to develop in a slab. Firstly, horizontal translational end restraint must be provided. Secondly, the total strain along a longitudinal fibre at the depth of the horizontal restraint must be non-zero if the restraint did not exist. From these conditions it is clear that there are only rare cases where membrane forces will *not* develop in a slab with horizontal restraint. For example, a slab with fully fixed ends, a uniform cross-section and a homogeneous material (ie uncracked slab) will not develop membrane forces. Also, a slab where the horizontal restraint only exists at the neutral axis of the slab will not develop membrane forces.

These requirements are illustrated in Figure 1, which shows a cross-section of a beam-and-slab bridge deck. Compressive membrane action develops transversely in the slab because cracks develop at mid-span and at the slab ends. This causes an extension in the longitudinal fibres of the slab, which is restricted by the horizontal restraint. In this case, the beams and the concrete in adjacent slabs provide the horizontal restraint.

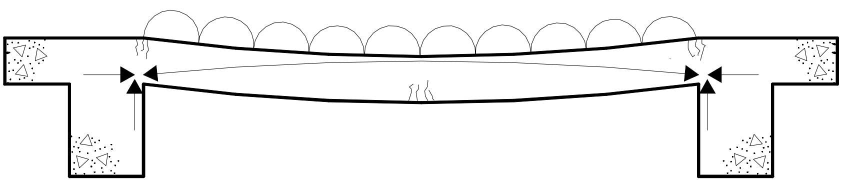

Figure 1: Diagrammatic representation of compressive membrane action

The existence of compressive membrane action in horizontally restrained slabs is well accepted by researchers and many previous investigations have been carried out in this area. Ockleston (1) was one of the earliest researchers to examine the behaviour of slabs under compressive membrane action. He load tested a number of slabs in a reinforced concrete building in South Africa and found that their strength was significantly higher than expected. In a later paper Ockleston (2) attributed this strength enhancement to the existence of compressive membrane action. Wood (3) and Park (4-6) were two of the earliest researchers to develop analytical methods to assess the strength of reinforced concrete slabs where compressive membrane action exists. The paper by Braestrup (7) provides an historical review of the analyses that have been developed by numerous researchers investigating this phenomenon. The theoretical formulations have generally been based on plasticity theory. Because of the complexity of this approach, many assumptions and simplifications have been required in those analyses. This lead to formulations that, while being fundamentally and theoretically important, did not provide accurate predictions of the strength and loading behaviour of reinforced concrete slabs under compressive membrane action. Because of this, simplified methods, such as those by Kirkpatrick et al (8), Rankin and Long (9) and Eyre (10) have recently been developed. To use these methods in practical situations, knowledge of the surround stiffness that exists for the slab is required, and this remains the greatest impediment to the application of the theory in bridge design and assessment.

This paper details a practical design approach to account for the enhancing effects of compressive membrane action. The surround stiffness is approximated from linear-elastic beam models and is then used in the method derived by Rankin and Long (9) to assess the strength of slabs where compressive membrane action is present.

Although advantage can be taken of compressive membrane action during the design of new structures, it is in the assessment of existing structures that the greatest benefit can be gained. There are existing reinforced concrete bridge decks in Australia that have inadequate ultimate strength when assessed for increased axle loads using elastic analysis and ultimate section capacity. In many cases the serviceability performance of these deck slabs will be satisfactory, and the only requirement is to increase the ultimate strength. A common response to this problem is to strengthen the deck slab by applying carbon fibre strips to the slab soffit. An alternative approach is to make an assessment of the ultimate strength of the deck slab allowing for the beneficial effect of compressive membrane action. By obtaining a more accurate assessment of the true strength of the deck slab, the need for strengthening may be removed.

2 TREATMENT IN DESIGN CODES

In design and assessment codes, the flexural strength of reinforced concrete slabs is usually determined using an elastic analysis and the ultimate strength of the cross-section. The yield-line theory developed by Johansen (11) can also be used to determine the ultimate strength of a reinforced concrete slab. These theories have a common limitation in that compressive membrane action within the slab cannot be taken into account.

The presence of compressive membrane action has been recognised for many decades now, but its obvious benefits are not fully utilised in design and assessment codes. The Ontario Highway Bridge Design Code first introduced provisions for compressive membrane action in 1979. This code (and later editions including the 2000 Canadian Highway Bridge Design Code) contain an empirical deck slab design method for slab on girder bridges that reduces the amount of reinforcing steel required in concrete deck slabs. The code allows this to be done only if certain restrictions on span length, slab thickness, detailing of diaphragms and connection between slab and beam are met.

In 2002 the United Kingdom Highways Agency published guidelines for the use of compressive membrane action in bridge decks. This allows for both simplified and rigorous analysis to utilise the beneficial effects of membrane action where adequate deck slab restraint exists.

2 ARCHING ACTION THEORY

McDowell McKee and Sevin (12) developed a theoretical model to describe the arching action of masonry walls. This model, which was adapted by Rankin and Long (9) to describe compressive membrane action in slabs, is discussed below.

Figure 2 illustrates the idealised model. It consists of a slab constrained between two rigid supports. The slab has depth of 2d and span of 2L. It is assumed that the slab can only transfer compressive stress to the restraint. The deformation of the slab is idealised as the rotation of two rigid bodies, about the centre and each end of the span. The rotation is assumed to occur about the point of contact, point A in Figure 3.

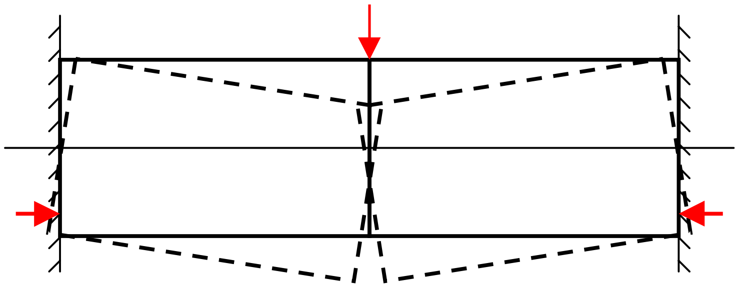

Figure 2 Idealised model of the slab

Figure 3 develops the relationships to define the strains on the contact surfaces at the end of each half of the slab. Because of symmetry, it is assumed that the strain distributions against the end support and at midspan are identical.

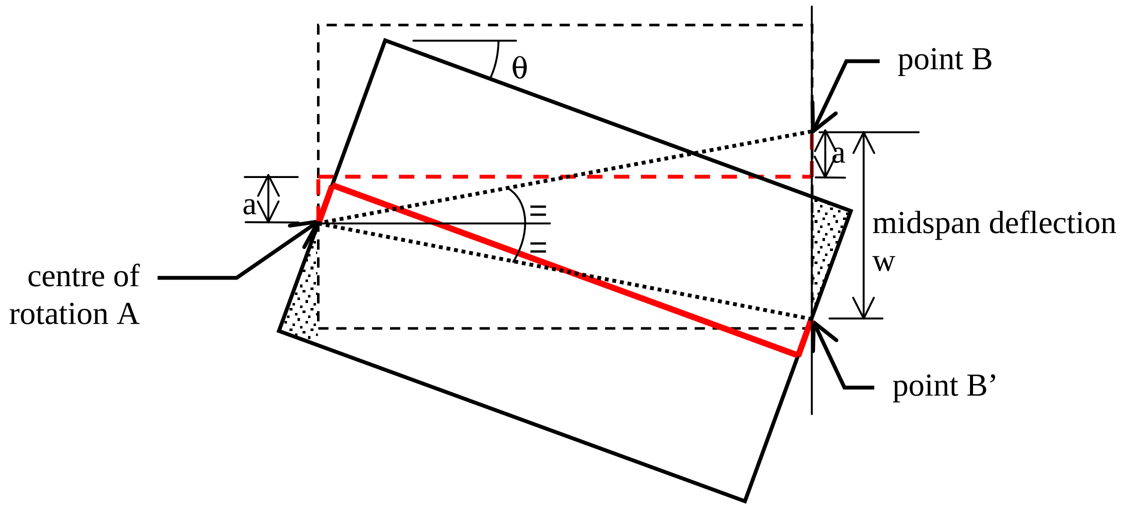

Figure 3 Geometry of the displacements

In Figure 3 the point B' defines the end of the contact surface at midspan. Therefore B' must lie on the centreline of the span, and the displacement of B to B' defines the midspan deflection. Furthermore, as B and B' are in the same vertical plane, the height of the centre of rotation, A, must be midway between B and B'. Therefore,

$$w = 4a \tag{1}$$

From Figure 4, the shortening of the bottom fibre of the slab at the contact surface, ΔL is given by,

$$\Delta L = (d - a)\tan\theta$$

and taking the approximation that $\tan\theta = \frac{w}{L}$,

$$\Delta L = (d - a)\frac{w}{L} = \left(d - \frac{w}{4}\right)\frac{w}{L}$$

Considering the bottom fibre of the slab over half the span, the strain is zero at midspan, so assuming a linear variation with the strain over the length of the half span, L, the strain in the bottom fibre at the contact surface is taken to be,

$$\varepsilon_b = 2 \times \frac{\Delta L}{L} = \frac{2w}{L^2}\left(d - \frac{w}{4}\right) \tag{2}$$

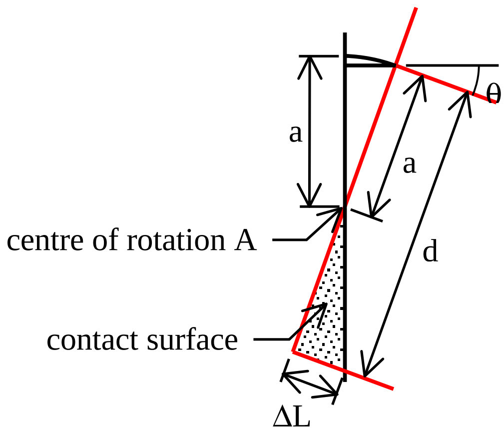

Figure 4 Geometry of the contact surface

3 APPLICATION TO CONCRETE SLABS

Rankin and Long (9) applied the arching theory of McDowell McKee and Sevin (12) to concrete slabs. To calculate the force that acts on the contact surface it is necessary to relate the strain from equation (2) to the stress in the concrete. An elastic-plastic stress strain curve was assumed, as shown in Figure 5.

As described by Rankin and Long (9), the value of the plastic strain, ε_c, is given by,

$$\varepsilon_c = \left(-400 + 60 f'_c - 0.33 f'^{\,2}_c\right) \times 10^{-6} \tag{3}$$

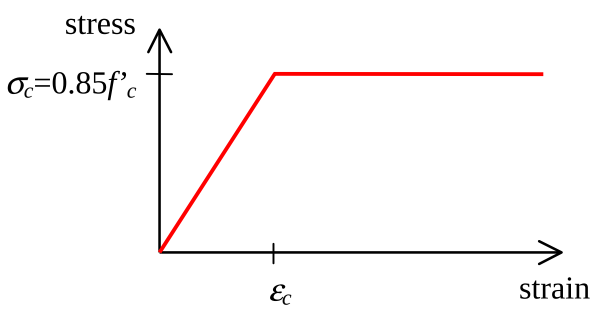

Figure 5 Assumed stress strain curve for concrete

Because of the assumption of an elastic plastic stress strain curve, the stress diagram on the contact face will take one of two forms as shown in Figure 6.

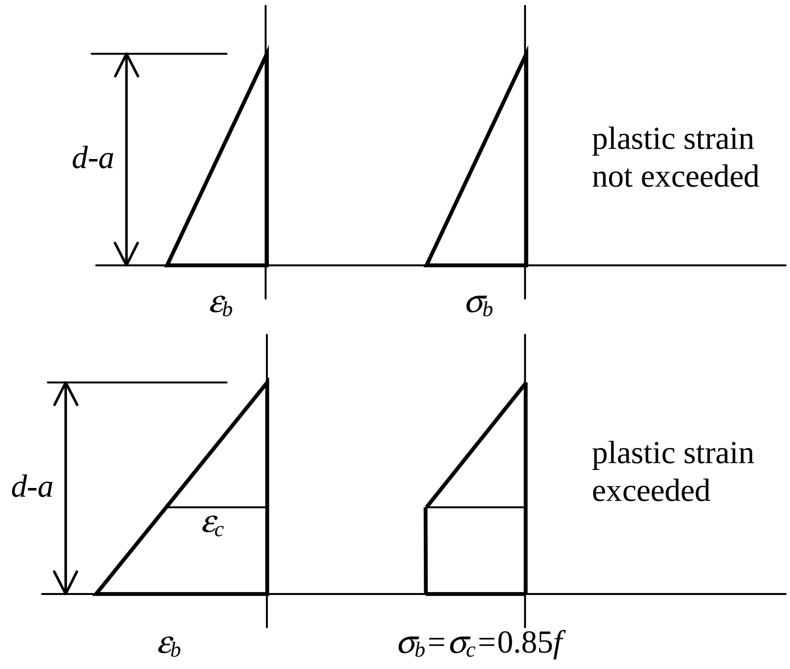

Figure 6 Stress diagrams on the contact surface

3.1 Plastic strain not exceeded

If the plastic strain is not exceeded, the force (per unit width of slab) on the contact surface is given by,

$$P = \frac{\varepsilon_b}{2} \times \left(\frac{\sigma_c}{\varepsilon_c}\right) \times (d-a) = \left(\frac{\sigma_c}{\varepsilon_c}\right) \frac{w}{L^2} \left(d - \frac{w}{4}\right)^2 \tag{4}$$

The lever arm between the two horizontal forces acting on half the slab can be approximated for small rotation θ as in Figure 7.

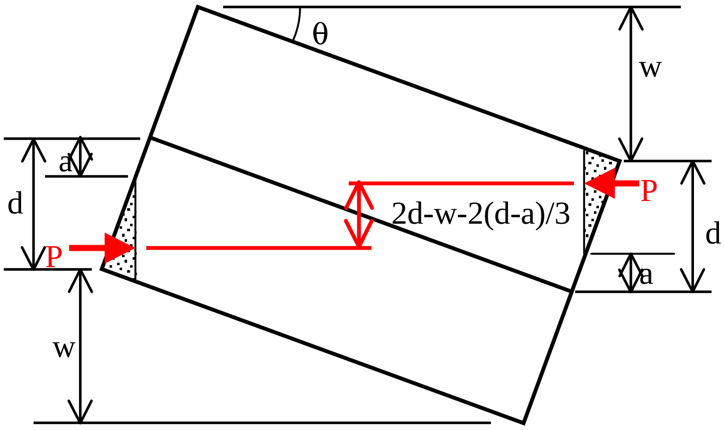

Figure 7 Lever arm for triangular stress distribution

Therefore the bending moment due to the compressive membrane action is given by,

$$
\begin{aligned}
M &= \left(\frac{\sigma_c}{\varepsilon_c}\right)\frac{w}{L^2}\left(d-\frac{w}{4}\right)^2\left(2d-w-\frac{2}{3}(d-a)\right) \\
&= \left(\frac{\sigma_c}{\varepsilon_c}\right)\frac{w}{L^2}\left(d-\frac{w}{4}\right)^2\left(\frac{4}{3}d-\frac{5}{6}w\right)
\end{aligned} \tag{5}
$$

The deflection that causes the maximum moment is found by differentiating equation (5) with respect to *w* which gives,

$$
\begin{aligned}
&(16d-4w)\left(8d^2-16w+5w^2\right)=0 \\
&\therefore w=\frac{8-2\sqrt{6}}{5}d \approx 0.62d
\end{aligned} \tag{6}
$$

Substituting $w = 0.62d$ into equation (2),

$$
\varepsilon_{b\max} = \frac{1.05d^2}{L^2} \tag{7}
$$

For a triangular stress distribution we require $\varepsilon_{b\max} < \varepsilon_c$, therefore the condition for a triangular stress distribution is,

$$
\frac{L}{d} > \sqrt{\frac{1.05}{\varepsilon_c}} \tag{8}
$$

Substituting $w = 0.62d$ into equation (5),

$$
M_{\max} = 0.362\left(\frac{\sigma_c}{\varepsilon_c}\right)\left(\frac{d^4}{L^2}\right) \tag{9}
$$

3.2 Plastic strain is exceeded

When $\frac{L}{d} < \sqrt{\frac{1.05}{\varepsilon_c}}$ the stress distribution is no longer triangular. At a distance *y* below the centre of rotation A (Figure 3), the change in length of a longitudinal fibre is,

$$\Delta L = y \tan\theta = y\frac{w}{L}$$

Therefore the strain at distance *y* below the centre of rotation A,

$$\varepsilon = 2y \times \frac{w}{L^2}$$

Setting $\varepsilon = \varepsilon_c$, gives the distance *y* to the end of the triangular distribution as,

$$y = \frac{\varepsilon_c L^2}{2w}$$

From Figure 8, the forces (per unit width of slab) on the contact surface are given by,

$$P_1 = \frac{\sigma_c}{2} \times \frac{\varepsilon_c L^2}{2w} \qquad \text{and} \qquad P_2 = \sigma_c \times \left(d - \frac{w}{4} - \frac{\varepsilon_c L^2}{2w} \right) \tag{10}$$

and the lever arms for the respective forces are,

$$\frac{2\varepsilon_c L^2}{3w} - \frac{w}{2} \qquad \text{and} \qquad \frac{\varepsilon_c L^2}{2w} d - \frac{3w}{4} \tag{11}$$

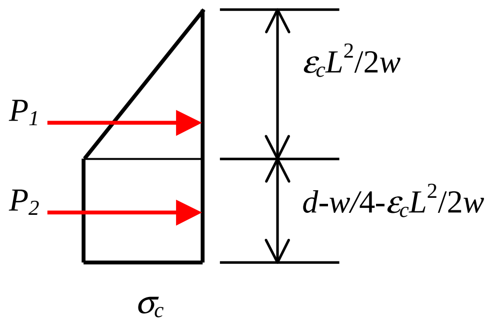

Figure 8 Stress and force when plastic strain is exceeded

Therefore the bending moment due to the compressive membrane action is given by,

$$\begin{aligned} M &= \left(\frac{\sigma_c}{2} \times \frac{\varepsilon_c L^2}{2w} \right)\left(\frac{2\varepsilon_c L^2}{3w} - \frac{w}{2} \right) + \sigma_c \times \left(d - \frac{w}{4} - \frac{\varepsilon_c L^2}{2w} \right)\left(\frac{\varepsilon_c L^2}{2w} d - \frac{3w}{4} \right) \\ &= \sigma_c \left(-\frac{\varepsilon_c{}^2 L^4}{12w^2} + \frac{\varepsilon_c L^2}{8} + d^2 - dw + \frac{3w^2}{16} \right) \end{aligned} \tag{12}$$

The deflection that causes the maximum moment is found by differentiating equation (12) with respect to *w* which gives,

$$\frac{\varepsilon_c{}^2 L^4}{6w^3} - d + \frac{3w}{8} = 0 \tag{13}$$

It can be seen that the maximum moment depends not only on *w*, but also on ε_c and *L*. Rankin and Long (9) introduced the non-dimensional parameter,

$$R = \frac{\varepsilon_c L^2}{4d^2} \tag{14}$$

whereby equation (13) can be rewritten as,

$$\frac{64R^2}{3} / \left(\frac{w}{d}\right)^3 - 8 + 3\left(\frac{w}{d}\right) = 0 \tag{15}$$

By varying R and calculating the maximum moment for each value of R, Rankin and Long (9) obtained the following equation as a curve fit to the maximum moment values,

$$M_{\max} = \sigma_c d^2 \left(1.08 - 4.03\sqrt{0.00033 + 0.1243R}\right) \tag{16}$$

3.3 Equation summary

If $\frac{L}{d} \geq \sqrt{\frac{1.05}{\varepsilon_c}}$, $\quad M_{\max} = 0.362\left(\frac{\sigma_c}{\varepsilon_c}\right)\left(\frac{d^4}{L^2}\right)$

If $\frac{L}{d} < \sqrt{\frac{1.05}{\varepsilon_c}}$, $\quad M_{\max} = \sigma_c d^2 \left(1.08 - 4.03\sqrt{0.00033 + 0.1243R}\right)$

where $R = \frac{\varepsilon_c L^2}{4d^2}$

$\sigma_c = 0.85 f'_c$

$\varepsilon_c = \left(-400 + 60 f'_c - 0.33 f'^{\,2}_c\right) \times 10^{-6}$

4 SOLUTION FOR NON RIGID RESTRAINT

Rankin and Long (9) dealt with non rigid restraints by considering them as elastic springs. The stiffness of a three pinned arch spanning $2L$, having elastic modulus E, area A, and spring restraints of stiffness K, will be the same as that of an arch with rigid restraints spanning a greater distance, $2L_r$, where,

$$L_r = L\left(\frac{EA}{KL} + 1\right)^{\frac{1}{3}} \tag{17}$$

For the same deflection, the bending moment due to the load on the elastically restrained arch is related to the bending moment on the rigidly restrained arch by,

$$M_{elastic} = M_{rigid} \frac{L}{L_r} \tag{18}$$

Applying the three pinned arch analogy to a slab with compressive membrane action, the maximum moment due to compressive membrane action in an elastically restrained slab is given by,

$$M_{elastic} = M_{\max} \frac{L}{L_r} \tag{19}$$

where $M_{\max}$ is from Section 3.3 above. In calculating L_r from equation (17), E should be the short term elastic modulus (as the method applies only to short term loading), and A is taken by Rankin and Long (9) to be the area of *half* the available slab depth (refer Section 5).

5 DEPTH AVAILABLE FOR COMPRESSIVE MEMBRANE ACTION

In the approach taken by Rankin and Long (9) the strength contribution from compressive membrane action is added to the flexural strength. Therefore the area of concrete available for compressive membrane action is reduced to allow for the compression required to balance the tension in the reinforcement. Therefore the depth of the slab that is available for compressive membrane action is taken as,

$$d_1 = 0.5\left(h - \left(\rho + \overline{\rho}\right)\frac{f_y d_{eff}}{0.85 f'_c}\right) \tag{20}$$

6 LATERAL RESTRAINT STIFFNESS

The application of equation (17) requires a value for the lateral restraint stiffness, *K*. Hon, Taplin and Al-Mahaidi (13-15) have developed a method for estimating the lateral restraint stiffness based upon a simple frame model of the slab. The axial stiffness of the slab surround is modelled as a number of springs, with the stiffness of each spring dependent on the width of the slab deck that each spring is modelling. The edge beam members are given the dimensions of the actual edge beam and adjacent slab (if it exists). A horizontal unit load is then applied to the edge beam and the displacement obtained using a linear-elastic analysis of the beam model. From this, the horizontal translational restraint stiffness can be determined. The method is illustrated in Figure 9.

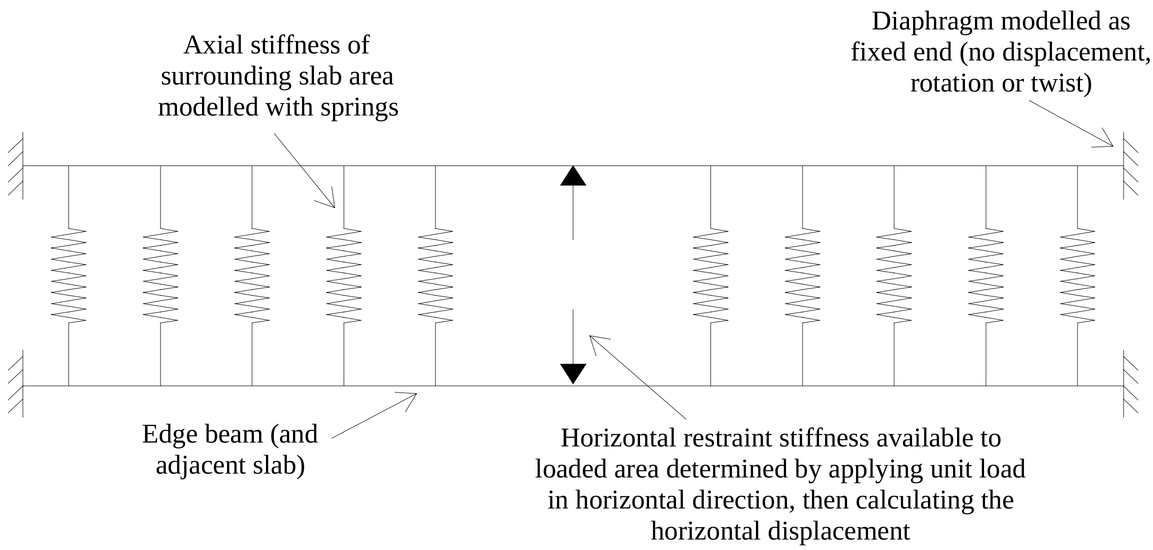

Figure 9 Frame analysis of the slab to determine the restraint stiffness

7 COMPARISON WITH MEASURED CAPACITIES

Tests were conducted by Hon, Taplin and Al-Mahaidi (13-15) to measure the load carried by slab decks (Figure 10). The results from those tests are presented in Figure 11. The results show that the slabs have significantly greater strength than predicted by flexural theory ignoring compressive membrane action. The results also show that the method described herein for including the contribution from compressive membrane action gives acceptable predictions of the measured strengths.

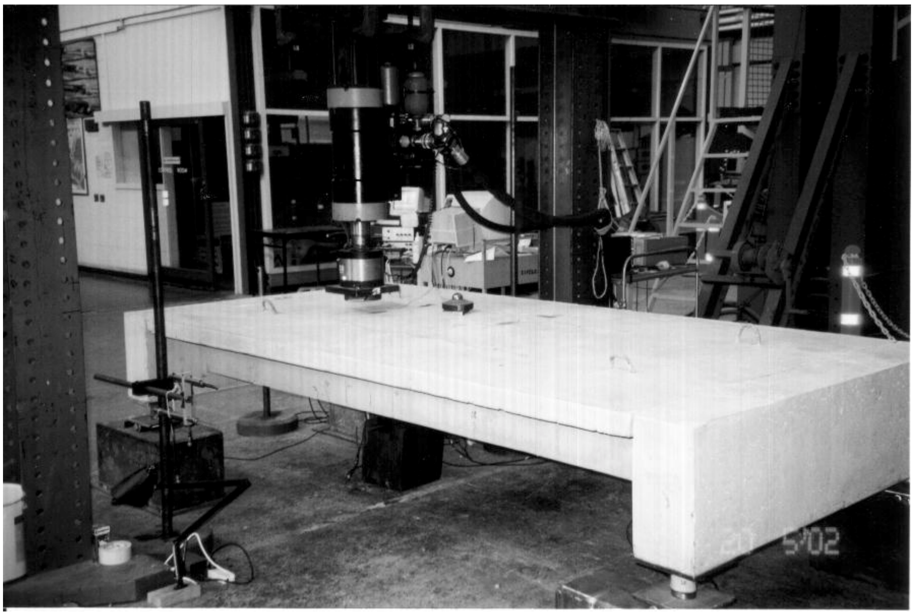

Figure 10 Testing of a slab deck

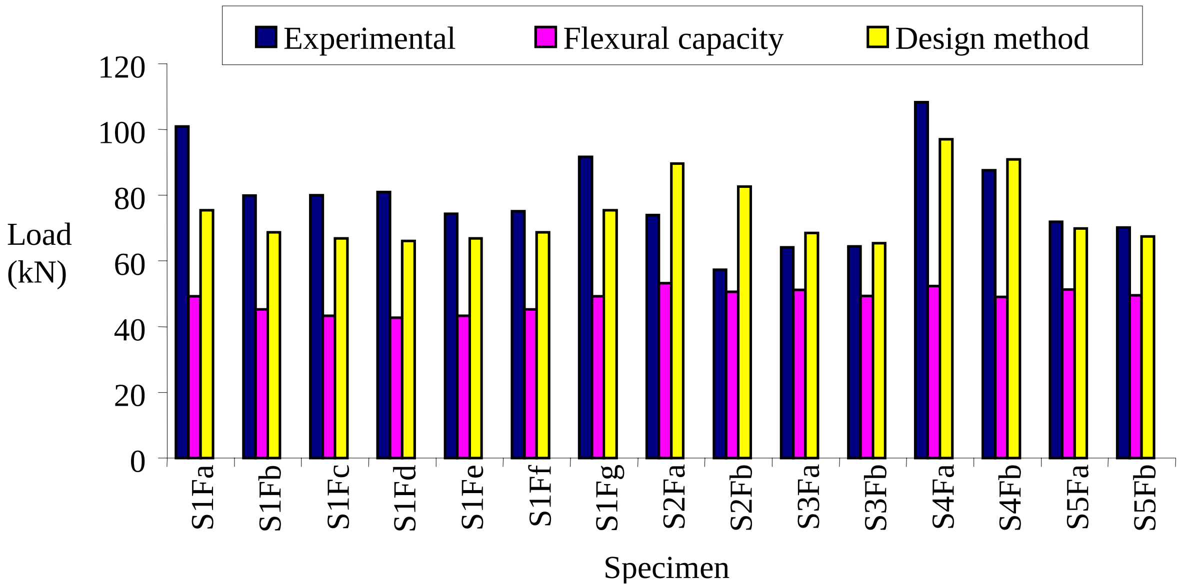

Figure 11 Comparison of slab strength results

8 CONCLUSIONS

A method has been described for including compressive membrane action in the strength assessment of bridge deck slabs. The method combines the approach adopted by Rankin and Long (9) with a simple approach developed by Hon, Taplin and Al-Mahaidi (13-15) to assess the lateral restraint stiffness provided by the surrounding beams and slab.

The application of the method has been compared to the results of an experimental investigation of the strength of deck slabs, and has given reasonable agreement with the measured strengths.

9 NOTATION

a	distance from mid depth of the slab to the centre of rotation (Fig 2, 3)
d	half the slab depth
d_1	depth of slab available for compressive membrane action
d_{eff}	effective depth of the slab
f'_c	concrete cylinder compressive strength
f_y	yield stress of the reinforcing steel
h	overall slab thickness
L	half the slab span
L_r	half the span of the equivalent rigidly restrained slab
M_{max}	moment capacity due to compressive membrane action
$M_{elastic}$	moment capacity due to compressive membrane action with elastic restraints
M_{rigid}	moment capacity due to compressive membrane action with rigid restraints
R	non dimensional parameter
w	midspan deflection
y	depth from the centre of rotation to the end of the triangular stress distribution
$\overline{\rho}$	negative reinforcement ratio
θ	angle of rotation of the half span
ρ	positive reinforcement ratio
ε_b	strain in the bottom fibre of the slab at the contact surface
σ_b	stress in the bottom fibre of the slab at the contact surface
ε_c	plastic strain of the concrete
σ_c	plastic stress in the concrete = $0.85f'_c$

10 REFERENCES

1 OCKLESTON, A. J. “Load tests on a three storey reinforced concrete building in Johannesburg” *The Structural Engineer* Vol. 33 1955 pp. 304-322.

2 OCKLESTON, A. J. “Arching action in reinforced concrete slabs” *The Structural Engineer* Vol. 36(6) 1958 pp. 197-201.

3 WOOD, R. H. “Plastic and elastic design of slabs and plates” London, Thames and Hudson. 1961

4 PARK, R. “Ultimate strength of rectangular concrete slabs under short-term uniform loading with edges restrained against lateral movement” *Proceedings of the Institution of Civil Engineers* Vol. 28 1964 pp. 125-150.

5 PARK, R. “The ultimate strength and long-term behaviour of uniformly loaded, two-way concrete slabs with partial lateral restraint at all edges” *Magazine of Concrete Research* Vol. 16(48) 1964 pp. 139-152.

6 PARK, R. “The lateral stiffness and strength required to ensure membrane action at the ultimate load of a reinforced concrete slab-and-beam floor” *Magazine of Concrete Research* Vol. 17(50) 1965 pp. 29-38.

7 BRAESTRUP, M. W. “Dome effect in RC slabs: Rigid-plastic analysis” *Journal of the Structural Division*, ASCE Vol. 106(ST6) 1980 pp. 1237-1253.

8 KIRKPATRICK, L, RANKIN, G. 1. B. and LONG, A. E. “Strength evaluation of M-beam bridge deck slabs” The Structural Engineer Vol. 6213(3) 1984 pp. 60-68.

9 RANKIN, G.I.B and LONG, A.E. “Arching strength enhancement in laterally-restrained slab strips” *Proceedings of the Institution of Civil Engineers*, *Structures and Buildings* Vol 122 1997 pp 461-467
10 EYRE, J.R. “Direct assessment of safe strengths of RC slabs under membrane action” *Journal of Structural Engineering* Vol 123(10) 1997 pp 1331-1338
11 JOHANSEN, K.W. “Yield-line theory” London, Cement and Concrete Association 1962
12 MCDOWELL, E.L., MCKEE, K. E. and SEVIN, E. “Arching action theory of masonry walls” Journal of the Structural Division, ASCE Vol. 82(ST2) 1956 pp. 915-1 - 915-18.
13 HON, A., TAPLIN, G. and. AL-MAHAIDI, R. “Compressive membrane action in reinforced concrete one way slabs”, *8th East Asia-Pacific Conference on Structural Engineering & Construction*, Singapore, December 2001
14 HON, A., TAPLIN, G. and. AL-MAHAIDI, R. “Investigating the behaviour of T-beam bridge decks in flexure” *Australasian Structural Engineering Conference*, Gold Coast, April 2001
15 HON, A., TAPLIN, G. and. AL-MAHAIDI, R. “Assessing the strength of reinforced concrete beam-and-slab bridge decks using compressive membrane action” *Concrete in the Third Millennium Conference*, Concrete Institute of Australia, 17-19 July 2003, Brisbane Australia